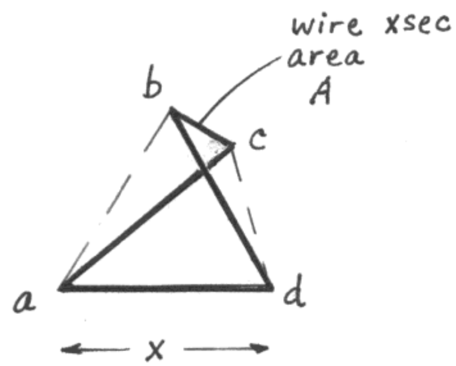
University of California, Berkeley
Physics H7B Spring 1999 (*Strovink*)

FINAL EXAMINATION

Directions. Do all six problems (weights are indicated). This is a closed-book closed-note exam except for three $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he or she will not give hints and will be obliged to write your question and its answer on the board. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

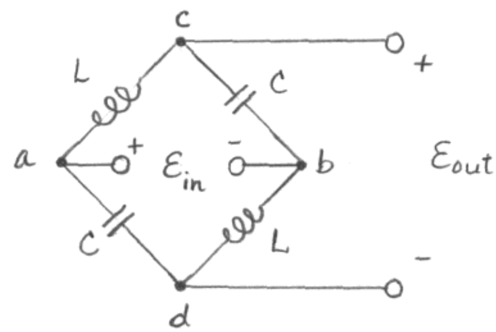
Problem 1. (35 points)
Four straight stainless steel wires of length x, cross-sectional area A, and resistivity ρ are welded together so that they lie along four of the six edges of a regular tetrahedron, as shown.



The remaining two sides ab and cd are empty. Consider a and b to be electrical input terminals, and c and d to be electrical output terminals.

a. (15 points)
When c is shorted to d, what resistance is measured between a and b?

b. (20 points)
Suppose that wires ac and bd are replaced by two identical inductors L, and wires bc and ad are replaced by two identical capacitors C. Across terminals a and b is placed a source of input EMF $\mathcal{E}_{\rm in}(t) = \mathcal{E}_0 \cos\omega t$ where $\mathcal{E}_0$ and ω are real constants. Across terminals c and d, an output EMF $\mathcal{E}_{\rm out}(t)$ is measured by an ideal voltmeter which draws no current.



Is there a value of ω for which the voltmeter will measure $\mathcal{E}_{\rm out}(t) = 0$? Explain.

Problem 2. (30 points)
Experiments using heavy electrons ("muons") have exploited the fact that, when a pi meson ("π^+") decays at rest into a muon ("μ^+") plus a neutrino ("ν"), the muon has a unique momentum with which its spin angular momentum is fully aligned. The reaction is

$$\pi^+ \rightarrow \mu^+ + \nu \ .$$

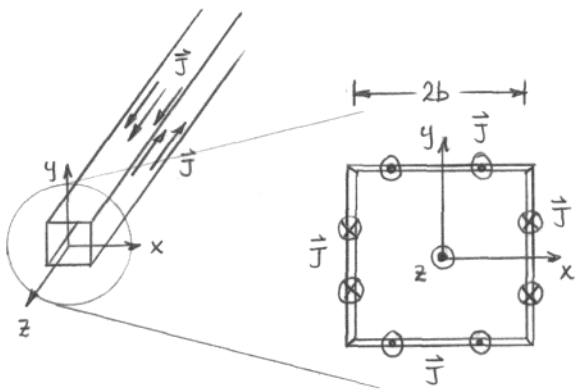
A beam of such muons is called a *surface muon beam* (because the pion is stopped near the surface of a solid target where it was produced by protons from a cyclotron). The muons in a surface beam are so well defined that, if they were allowed to impinge normally on a book, nearly all would stop in the same page.

Given that the muon mass is 3/4 of the pion mass, while the neutrino mass is negligibly small, compute the velocity of the muons in a surface beam, expressed as a fraction of the speed of light.

Problem 3. (35 points)
A *Panofsky quadrupole* magnet consists of four long thin copper bars, pointing in the $\hat{\mathbf{z}}$ direction (out of the page), arranged so that their inside surfaces form a square box of side $2b$. The bars at $y = \pm b$ carry a uniform current density in the $+\hat{\mathbf{z}}$ direction while the bars at $x = \pm b$ carry the same current in the $-\hat{\mathbf{z}}$ direction. Within the box enclosed by the bars, the magnetic vector potential is

$$\mathbf{A} = \frac{\alpha}{2}\hat{\mathbf{z}}(y^2 - x^2) ,$$

where α is a constant.



a. (10 points)
Suppose that a particle of charge $+e$ travels along $\hat{\mathbf{z}}$ at position $(x, y) = (0, y)$. Show that the particle is deflected toward (0,0) with a force that is proportional to y. (This means that, in the y projection, the Panofsky quadrupole acts as a *converging lens*. However, it acts as a *diverging lens* in the x projection. Fortunately, the combination of a converging and a diverging lens of equal strength remains slightly converging, if the two lenses are separated along their axis; this allows a pair of quadrupole magnets to focus a particle beam weakly in both the x and y projections. One of the first experiments to use this fact discovered the antiproton at the Berkeley Bevatron in 1956.)

b. (10 points)
Prove that the current density $\mathbf{J}$ within the box enclosed by the bars ($|x| < b$ and $|y| < b$) must be zero. (This allows the box to be evacuated so that a particle beam can travel unimpeded within it.)

c. (15 points)
Suppose that a different region of space has

$$\mathbf{B}(x, y) = A_0(\hat{\mathbf{x}}y - \hat{\mathbf{y}}x) ,$$

where A_0 is a constant (a "bullseye" magnetic field). Show that the current density along $\hat{\mathbf{z}}$ must be nonzero everywhere in the region; give its magnitude and any dependence that it may have on x and y. (This magnetic field acts as a converging lens in *both* the x and y projections. However, the beam particles are required to pass through the magnet's current-carrying element, which needs to be made as light as possible, e.g. of molten lithium.)

Problem 4. (35 points)
Consider a uniform region of space containing an insulating material with fixed dielectric constant ϵ and magnetic permeability μ.

a. (3 points)
Write Faraday's law in differential form.

b. (3 points)
For this material there are two differential versions of Ampere's Law, as modified by Maxwell – one version uses the free current density $\mathbf{J}_{\text{free}}$, the other uses the total current density. Write down the version that uses $\mathbf{J}_{\text{free}}$ (which is zero for this insulating material).

c. (3 points)
Expressing $\mathbf{H}$ in terms of $\mathbf{B}$ and μ, and $\mathbf{D}$ in terms of $\mathbf{E}$ and ϵ, taking advantage of the fact that ϵ and μ are constant in this material, rewrite equation (b.) in terms of $\mathbf{B}$ and $\mathbf{E}$.

d. (3 points)
Take $\frac{1}{c}\frac{\partial}{\partial t}$ of both sides of equation (c.). On the left-hand side, interchange the order of differentiation, i.e. apply $\frac{1}{c}\frac{\partial}{\partial t}$ to $\mathbf{B}$ before taking its curl.

e. (3 points)
Use equation (a.) to substitute for $\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}$. Now you should have an equation in which $\mathbf{E}$ is the only vector field that appears.

f. (3 points)
Use the identity

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E}$$

to eliminate $\nabla \times (\nabla \times \mathbf{E})$ from the left-hand side of equation (e.).

g. (6 points)
Give an argument, based on the absence of free charges in this insulator, and the strict proportionality of $\mathbf{E}$ to $\mathbf{D}$, which allows you to ignore one of the terms on the left-hand side of equation (f.).

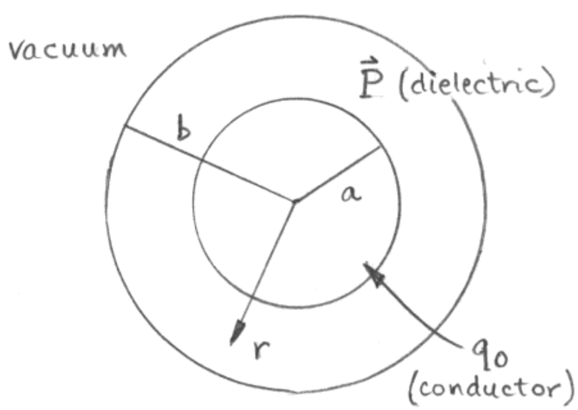
h. (6 points)
Your result should be a wave equation for $\mathbf{E}$. Show that any function of $(kx - \omega t)$, where k and ω are constants, solves this equation.

i. (5 points)
Calculate ω/k, the phase velocity of the solution (h.). Evaluate it in terms of c, ϵ, and μ.

Problem 5. (30 points)
An (insulating) hollow spherical shell of dielectric with inner (outer) radius a (b) has "frozen-in" polarization

$$\mathbf{P} = \hat{\mathbf{r}} \frac{q_0}{2\pi(a+b)r} ,$$

where r is the radius measured from its center and q_0 is a constant. The dielectric encloses a conducting sphere of radius a which holds total free charge q_0 (see the figure).



At what values of r does $\mathbf{E}$ vanish? [Your answer may include particular values of r (including those which are not finite), and/or ranges of r.] Note that this (nonlinear) dielectric's electric susceptibility is not defined or supplied here, and it should not appear in your answer.

Problem 6. (35 points)
Consider a hollow cubical box containing particles which make elastic collisions with its walls.

a. (10 points)
Suppose that the particles are molecules of a perfect gas. Using standard arguments about the number of molecules bouncing off an area of wall per unit time, and the momentum per collision that is imparted to the wall, prove that the pressure p (N/m^2) and the kinetic energy density u (J/m^3) of the gas are related by

$$p = \frac{2}{3} u .$$

b. (15 points)
Suppose that the box is filled not with molecules, but with electromagnetic radiation, which is quantized into photons. These photons can be considered to be massless particles which, like the perfect gas molecules, do not interact with each other and bounce elastically off the walls. Deduce the relationship between the pressure p and the energy density u of the electromagnetic radiation.

b. (10 points)
At sufficiently high temperature, the electromagnetic radiation pressure inside the box would be sufficient to balance the ambient pressure (10^6 dynes/cm^2) of the earth's atmosphere at sea level. If this were to occur, what would be the root mean square magnetic field (in gauss) inside the cavity?